Resonances from Higher Order S-Matrix Poles with Exponential Decay

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Abstract

In analogy to Gamow vectors describing resonance states from first order S-matrix poles, one can define Gamow vectors from higher order poles of the S-matrix. With these vectors we are going to discuss a density operator that describes exponentially decaying resonances from higher order poles.

Singularities of the analytically continued S-matrix have been used to describe resonances that decay exponentially [1]. But these states could not be described in ordinary Hilbert space quantum mechanics which doesn't provide the elements to describe the purely exponentially decaying vectors [2]. Resonances from higher order poles, in particular double poles, have also been mentioned [3,4], but were always associated with an additional polynomial time dependence [3]. However, operators in the form of finite dimensional matrices consisting of non-diagonalizable Jordan blocks have been discussed in connection with resonances numerous times in the past [5]

With the introduction of the rigged Hilbert space [6] it was possible to describe resonances [7–9], with the features usually attributed to resonances, i.e. exponential decay law and Breit-Wigner energy distribution. They are constructed from the first order poles of the analytically continued S-matrix on the second Riemann sheet of the complex energy plane, which come in pairs above and below the real axis. The pole at $z_R = E_R - i\Gamma/2$ corresponds to the decaying state defined for times $t \geq 0$, and the pole at $z_R^* = E_R + i\Gamma/2$ to the respective growing state for $t < 0$. The Gamow vectors are generalized eigenvectors of a self-adjoint Hamiltonian H, whose adjoint $H^\times$ in the rigged Hilbert space is an extension of the adjoint $H^\dagger = H$ in the Hilbert space, with complex eigenvalues $E_R \pm i\Gamma/2$ (energy and lifetime). These vectors form a complex basis system expansion, in which the Hamiltonian can be represented by a diagonal matrix with complex energies on the diagonal. The time evolution is given by a semigroup operator [10] which time translates the decaying state vectors for $t \geq 0$, and the growing state vectors for $t < 0$.

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The mathematical procedure by which these Gamow vectors were introduced suggests a straightforward generalization to higher order Gamow vectors which are derived from higher order poles of the S-matrix [11]. It can be shown that the r-th order pole of a unitary S-matrix leads to r generalized vectors of order $k = 0,\ 1,\ \ldots,\ r-1$, [12]

$$|z_R^-\rangle^{(0)}\,,\ |z_R^-\rangle^{(1)}\,,\ \cdots, |z_R^-\rangle^{(k)}\,, \cdots, |z_R^-\rangle^{(r-1)}\,. \tag{1}$$

In the same way one derives an analogous set of r generalized vectors for the S-matrix pole at z_R^* associated with the growing state. The higher order Gamow vectors form a complex basis vector expansion that spans an r-dimensional subspace $\mathcal{M}_{z_R}$ of the RHS. They are Jordan vectors [13] of degree $k+1$, and they are generalized eigenvectors [14] of a self-adjoint Hamiltonian with the complex eigenvalue $z_R = E_R - i\Gamma/2$ such that

$$\begin{aligned} H^\times|z_R^-\rangle^{(0)} &= z_R\,|z_R^-\rangle^{(0)} \\ H^\times|z_R^-\rangle^{(k)} &= z_R|z_R^-\rangle^{(k)} + k|z_R^-\rangle^{(k-1)};\ \ k = 1, \ldots, r-1\,. \end{aligned}$$

This means that $H^\times|_{\mathcal{M}_{z_R}}$ is represented by a finite dimensional Jordan block matrix of degree r.

The time evolution of the higher order Gamow vectors has a polynomial time dependence besides the exponential:

$$e^{-iH^\times t}|z_R^-\rangle^{(k)} = e^{-iz_R t}\sum_{\nu=0}^{k}\begin{pmatrix} k \\ \nu \end{pmatrix}(-it)^\nu\,|z_R^-\rangle^{(k-\nu)} \qquad t > 0\,. \tag{2}$$

The semigroup operator $e^{-iH^\times t}$ transforms between different $|z_R^-\rangle^{(k)}$ that belong to the same pole of order r at z_R, but it does not transform out of $\mathcal{M}_{z_R}$. A higher order Gamow vector of degree $k+1$ is transformed into a superposition of higher order Gamow vectors of the same and all lower degrees. The time evolution of the higher order Gamow vectors leads, as in the case of the ordinary Gamow vectors, to an intrinsic microphysical arrow of time [10].

The label k of the higher order Gamow vectors is not a quantum number in the usual sense. Basis vectors are usually labeled by quantum numbers associated with eigenvalues of a complete system of commuting observables ([7], chap. IV), but there is no physical observable that the label k is connected to. Therefore, the different $|z_R^-\rangle^{(k)}$ in the subspace $\mathcal{M}_{z_R}$ do not have a separate physical meaning.

In analogy to von Neumann's description of physical states by dyadic products of state vectors, Gamow states have been described by dyadic products of the ordinary Gamow vectors [7]. Examples of these states with their exponential decay law and their Breit-Wigner energy distribution have been observed in abundance as resonances and decaying states. Theoretically, there is no reason to exclude quasistationary

states from higher order poles. One argument made against their existence was the polynomial time dependence that was vaguely associated with them and which has not been observed.

For a microphysical decaying state associated with an $(n+1)$-st order S-matrix pole, the structure of the complex basis vector expansion [12] (pole term of the S-matrix element) suggests as a form of a higher order Gamow density operator:

$$W^{(n)} = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} |z_R^-\rangle^{(k)} \; {}^{(n-k)}\langle^- z_R| \, . \tag{3}$$

This microphysical state is a mixture of non-reducible components. In spite of the fact that the higher order Gamow vectors have an additional polynomial time dependence, this microphysical state obeys a purely exponential decay law [12],

$$W^{(n)}(t) = e^{-\Gamma t} \, W^{(n)}(0) \, ; \quad t \geq 0 \, . \tag{4}$$

However, they do not describe resonances with a simple Breit-Wigner energy distribution. Instead their energy distribution is a sum of the Breit-Wigner energy distribution and its derivatives up to order $n = r - 1$.It can be shown that these density operators are the only operators that can be constructed from the higher order Gamow vectors (describing resonances from higher order S-matrix poles) that lead to an exponential decay law [15]. In the zeroth order case one trivially deals with a pure state. However, for higher order states, "pure" has probably no meaning, since k is not a quantum number connected with a physical observable.

At presence there is little empirical evidence for the existence of these higher order pole states in nature. Our results suggest that the empirical objection to the existence of higher order poles of the S-matrix does not rule out the possibility of exponentially decaying states constructed from higher order Gamow vectors.

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